On the simple and partial Mantel tests with spatial data

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The (simple) Mantel test

Mantel N., **The detection of disease clustering and a generalized regression approach**, *Cancer Research*, 27, 209-220, 1967.

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- Goal: *"identifying subtle time-space clustering of disease, as may be occurring in leukemia"*
- Data: $(x_i, y_i)_{i=1,\dots,n}$ observations of a space-time point process
- Idea:
  - transform data so as to get two univariate variables
  - compute correlation of transformed data
  - assess significance of correlation by some permutation method

The simple Mantel test: detailed algorithm

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- Compute $D^x = (|x_i - x_j|)_{i,j}$ and $D^y = (|y_i - y_j|)_{i,j}$
- Compute the empirical correlation r between D^x and D^y
- For `iter =1,N`
  - draw a random permutation τ of $1, ..., n$
  - compute $D^x_\tau = (|x_{\tau(i)} - x_{\tau(j)}|)_{i,j}$
  - compute the empirical correlation r_τ between D^x_τ and D^y
- If $|r|$ larger than some quantile estimated from the r_τ values: report that there is *"subtle time-space clustering of disease"*

The partial Mantel test

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Smouse, P.E., J.C. Long, R.R. Sokal, **Regression and Correlation Extensions of the Mantel Test of Matrix Correspondence**, Systematic Zoology, 35(4), 627-632, 1986.

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- x_i and y_i observations of p and q variables for n statistical units.
- still attempts to assess the dependence between x and y
- need to *"filter out"* or *"control for"* the effect of a third variable z (e.g. z_i spatial coordinates of obs. i)

The partial Mantel test: detailed algorithm

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- Compute $D^x = (|x_i - x_j|)_{i,j}$, $D^y = (|y_i - y_j|)_{i,j}$ and $D^z = (|z_i - z_j|)_{i,j}$
- Compute residuals $\tilde{D}^x$ of linear regressions $D^x \sim D^z$
- Compute residuals $\tilde{D}^y$ of linear regressions $D^y \sim D^z$
- Compute the empirical correlation r between $\tilde{D}^x$ and $\tilde{D}^y$
- For `iter =1,N`
  - draw a random permutation τ of $1, ..., n$
  - compute $\tilde{D}^x_\tau$ as above for permuted x_i values
  - compute the empirical correlation r_τ between $\tilde{D}^x_\tau$ and $\tilde{D}^y$
- Assess significance of r by comparing to quantiles of r_τ.

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Features of the method

- deals with multivariate data
- synthetize data into a single numerical value
- does not seem to rely on any distributional assumption

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- Simple Mantel test [Mantel, 1967]: $\geq$ 5000 ISI citations
- Partial Mantel test [Smouse et al., 1986]: $\geq$ 1000 ISI citations
- Implemented in most ecology computer programs
- Countless number of articles using the Mantel tests citing other supporting references
- Routinely used in landscape genetics: x genotypes, y environmental variables, z geographical coordinates
- Practice strongly rooted:

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"Referee 3 pointed out some issues with the Mantel tests but they are so widely used in lansdcape genetics that this comment can be disregarded."

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More formal definition involves...

- A null hypothesis
- A method to derive a p-value
- Some additional distributional assumptions

Are the Mantel tests appropriate?

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A common implementation:

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A common implementation:

- x_i mutivariate genotype or phenotype.
  Due to population history and limited mixing in space x is spatially-autocorrelated
- y_i multivariate descriptor of landscape (elevation, temperature, vegetation cover).
  Due to bio/geo-physical laws y is spatially-autocorrelated
- Interest in testing H_0: x and y are independent

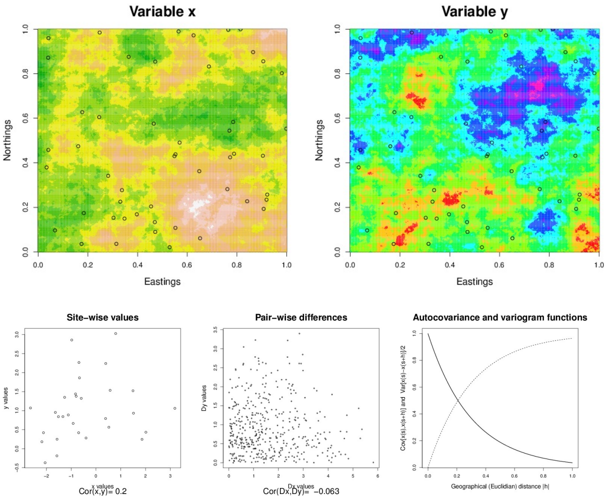
A simulation study

A simulation study

Simulation to mimic the situation of one phenotypic variable and one environmental variable.

- $s_1, ..., s_n$ n=50 sites in $[0,1]^2$
- $x(s_1), ..., x(s_n)$ values of a GRF with expo. covariance
- $y(s_1), ..., y(s_n)$ values of a GRF with expo. covariance
- x and y independent
- common scale param. κ

Example of simulated data



Simulation study (cont')

- simulation above repeated for 200 realizations of x and y
- p-values for simple Mantel test
- p-value for partial Mantel test with matrix D^s entered to "control the effect of space".
- common scale param. κ vaying from 0 to 0.7
- plot of ordered p-values against quantiles of a uniform distribution
- Under H_0, the p-values should be uniformly distributed [Schweder and Spjøtvoll, 1982]

Qq-plots of p-values obtained on simulated data

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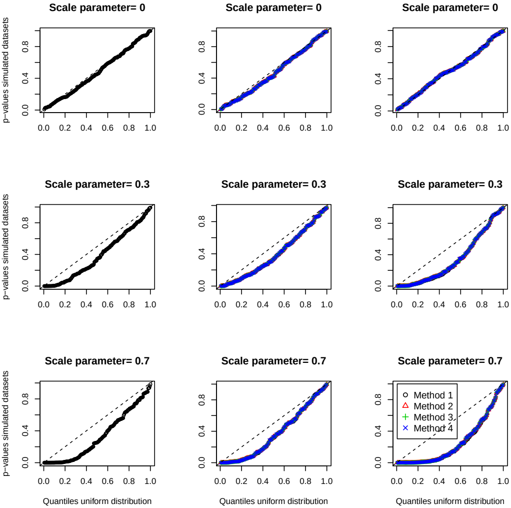

Figure: Left: simple Mantel test. Middle: partial Mantel test, no drift. Right: partial Mantel test, RFs with linear trend.

What's wrong with the Mantel tests?

What's wrong with the Mantel tests?

Mantel tests are based on permuation of one of the data vector entries

- Permutation of x values breaks the potential dependence between x and y
- Also breaks the spatial structure of x!!

The Mantel test fallacy:

$$cor(D^x_\tau, D^y) \overset{\mathcal{L}}{\neq} cor(D^x, D^y)$$

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  - lattice data: shift permutation

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  - lattice data: shift permutation
- Testing in a GLMM framework

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Thank you!

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