Aalto University
School of Science

9th SSIAB Workshop, Avignon - May 9-11, 2012

Testing of mark independence for marked point patterns

Mari Myllymäki

Department of Biomedical Engineering and Computational Science
Aalto University
mari.myllymaki@aalto.fi

Outline

The talk is based on the paper

P. Grabarnik, M. Myllymäki and D. Stoyan (2011). Correct testing of mark independence for marked point patterns. *Ecological Modelling* 222, 3888–3894.

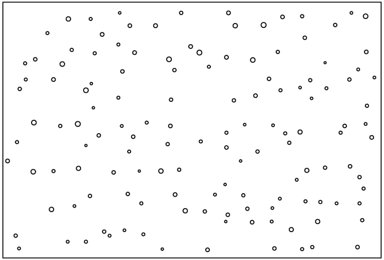
and discusses

- the conventional envelope test
- the refined envelope test
- the deviation test

through two marked point pattern data examples.

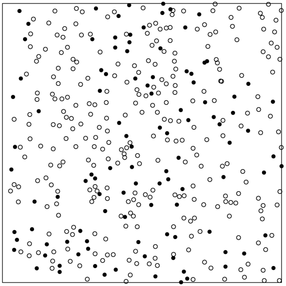
Data examples

Tharandter Wald: These data observed in a 56 m $\times$ 38 m rectangle come from a Norway spruce forest in Saxony (Germany).



Circles are proportional to the diameters of trees at breast height (=marks).

Frost shake of oaks: 392 oak trees observed in a 100 m $\times$ 100 m square at Allogny in France (Courtesy to Goreaud & Pelissier, 2003).



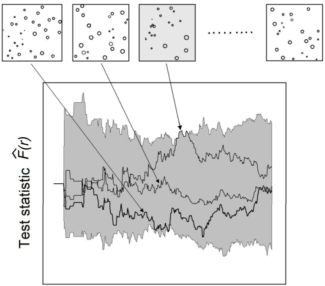
White circle = 1, a sound oak;
Black circle = 2, an oak suffering from frost shake

Our question here:

are the marks independently assigned for the points in an originally non-marked point pattern?

"Random labeling" hypothesis

How is the hypothesis typically tested?



Monte Carlo significance tests (Besag and Diggle, 1977)

- makes $s = 99$ simulations under the null hypothesis (How?)
- chooses a summary function $F(r)$ and calculated its estimate $\hat{F}(r)$ for data and each simulated marked point pattern
- Then either 1) calculates the minimum and maximum for each r in $[r_{\min}, r_{\max}]$

$$F_{\text{up}}(r) = \max_{i=2,\ldots,s+1} \hat{F}_i(r),$$

$$F_{\text{low}}(r) = \min_{i=2,\ldots,s+1} \hat{F}_i(r).$$

  and compared the data function to the envelopes, or,
  2) summarizes the information contained in the functional summary statistic $F(r)$ into a scalar test statistic

Consider first 1)!

The summary function?

Here the summary functions

$$L_{mm}(r) = \sqrt{\frac{K_{mm}(r)}{\pi}} \qquad \text{(Tharandter Wald data)}$$

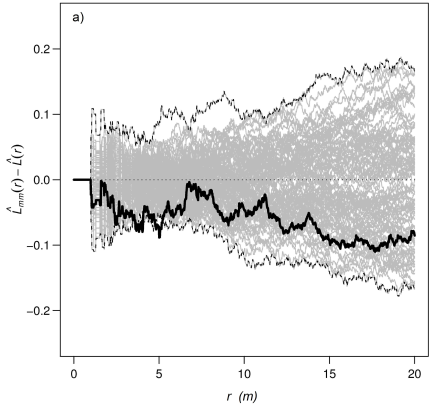
and

$$L_{12}(r) = \sqrt{\frac{K_{12}(r)}{\pi}} \qquad \text{(Frost shake of oaks data)}$$

are used, which both are generalizations of Ripley's K-function to marked or bivariate point patterns.

Envelopes for the Tharandter Wald data

$s = 99$



Conclusions?

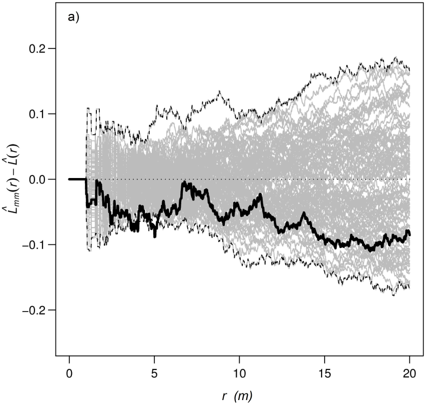
Problem of the envelope test

The spatial correlations are inspected for a range of distances simultaneously.

- Ripley (1977)
  - introduced envelope tests
  - mentioned that the frequence of committing the type I error in the envelope test may be higher than for a single distance test
- Diggle (1979, 2003)
  - proposed the deviation test
- Loosmore and Ford (2006)
  - adopted the deviation test
  - demonstrated the multiple testing problem of envelope test by estimating the type I error probability by simulation for the complete spatial randomness hypothesis based on the nearest neighbour distance distribution function
  - rejected the envelope test

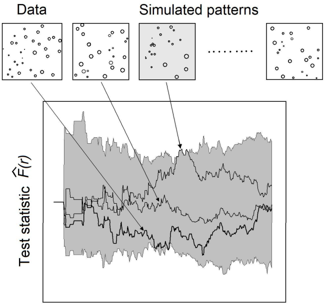
Envelopes for the Tharandter Wald data

$s = 99$, type I error approximation ≈ 0.48



Conclusions?

Type I error approximation?



In the case of minimum and maximum envelopes, the type I error is approximated by t/s where

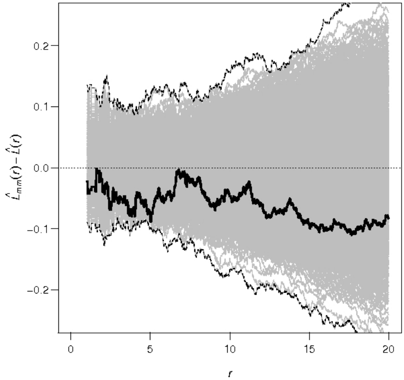
- t is the number of those simulations that take part in forming the envelopes
- s is the total number of simulations

Towards the refined envelope test

A natural way to make the envelope method valid, i.e. to obtain a reasonable type I error, is to increase the number of simulations from which the envelopes are calculated.

Envelopes for the Tharandter Wald data

$s = 1999$, type I error approximation ≈ 0.04



Conclusions?

The refined envelope test

The refined testing procedure = the envelope test, where

- the type I error probability is evaluated and taken into account in making conclusions
- if the choice of the number of simulations s leads to an unacceptably large type I error, s can be increased so that the type I error comes close to a desired value

The refined envelope test is then a rigorous statistical tool.

Deviation test

A deviation test

- summarizes information on $F(r)$ into a *single* number

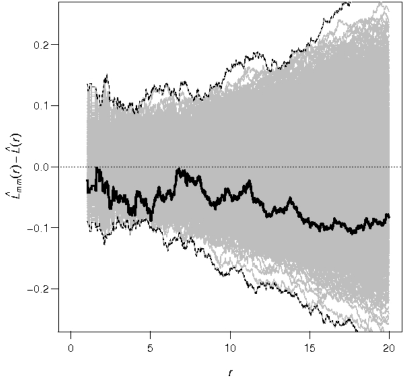
$$u_i = \max_{r_{\min} \leq r \leq r_{\max}} |\hat{F}_i(r) - F_{H_0}(r)|,$$

$$u_i = \int_{r_{\min}}^{r_{\max}} (\hat{F}_i(r) - F_{H_0}(r))^2 dr,$$

- is based on the rank of the data statistic
- provides the exact type I error probability, i.e. the null hypothesis is declared false, when it is true, precisely with the prescribed probability (Barnard, 1963; Besag &Diggle, 1977)

The data example 1

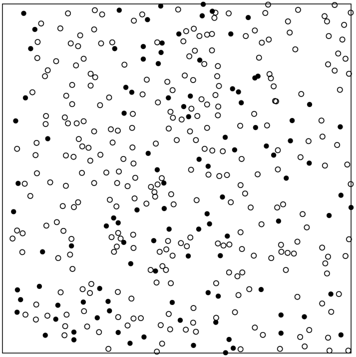
$s = 1999$, type I error approximation ≈ 0.04



Max-deviation: $\hat{p} = 0.31$; Int-deviation: $\hat{p} = 0.20$. Conclusions?

Data example 2

Frost shake of oaks: 392 oak trees observed in a 100 m × 100 m square at Allogny in France (Courtesy to Goreaud & Pelissier, 2003).



White circle = 1, a sound oak;
Black circle = 2, an oak suffering from frost shake

Data example 2

Earlier studies:
Goreaud & Pelissier (2003) and Illian et al. (2008):

- used the L_{12}-function and the envelope test
- G & P: 0.5%-lower and -upper envelopes based on $s = 10000$ simulations
- Illian et al.: minimum and maximum envelopes from $s = 99$ simulations
- came to the conclusion to reject the random labeling hypothesis

Data example 2

Earlier studies:

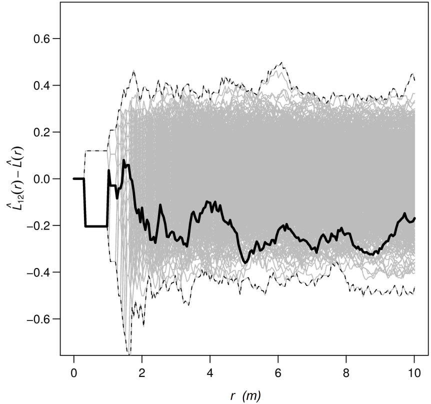
Goreaud & Pelissier (2003) and Illian et al. (2008):

- used the L_{12}-function and the envelope test
- G & P: 0.5%-lower and -upper envelopes based on $s = 10000$ simulations
- Illian et al.: minimum and maximum envelopes from $s = 99$ simulations
- came to the conclusion to reject the random labeling hypothesis

Type I error approximation: 1) 0.21 2) 0.41

Data example 2

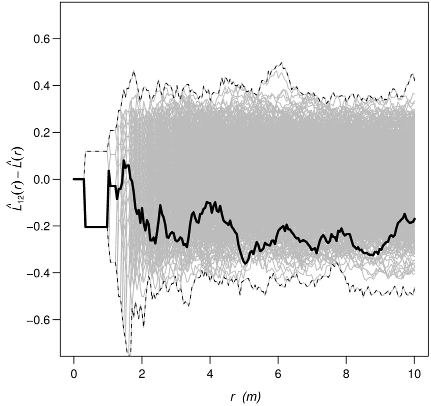
$s = 999$, type I error approximation ≈ 0.04



Conclusions?

Data example 2

$s = 999$, type I error approximation ≈ 0.04



Max-deviation: $\hat{p} = 0.20$; Int-deviation: $\hat{p} = 0.04$. Conclusions?

Discussion

The deviation test

- + do not need so many simulations
- + *p*-values can be easily estimated
- + different forms
- - says only little about the reason of rejection
- - says nothing on the scales at which there is behavior of $F(r)$ leading to rejection
- - performance depends on the behavior of the variance of $F(r)$ over the range of chosen distances ($\rightarrow$ more sophisticated edge correction methods, Ho & Chiu, 2006)

Discussion

The refined envelope test

- + help to detect reasons why the data contradict the null hypothesis (important when ecologists seek for alternative hypothesis!)
- + also raw estimators can be used (as long as the same estimator is used for $F_1(r)$ and $F_i(r)$, $i = 2, \ldots, s + 1$
- - needs many simulations
- -(?) no *p*-values

We recommend to couple formal testing with diagnostic tools using non-cumulative functions.

References

- Barnard, G.A., 1963. Discussion of paper by M.S. Bartlett. J. R. Stat. Soc. B25, 294.
- Besag, J., Diggle,P. J., 1977. Simple Monte Carlo tests for spatial pattern. Appl. Stat. 26, 327–333.
- Goreaud, F., Pélissier, R., 2003. Avoiding misinterpretation of biotic interactions with the intertype K12-function: Population independence vs. random labelling hypotheses. J. Veg. Science 14, 681–692.
- P. Grabarnik, M. Myllymäki and D. Stoyan. Correct testing of mark independence for marked point patterns. *Ecol. Mod.* 222, 3888–3894.
- Ho, L.P., Chiu, S.N., 2006. Testing the complete spatial randomness by Diggle's test without an arbitrary upper limit. J. Stat. Comp. Simul. 76, 585–591.
- Illian, J., Penttinen, A., Stoyan, H., Stoyan, D., 2008. Statistical Analysis and Modelling of Spatial Point Patterns. Wiley, Chichester.
- Loosmoore, N.B., Ford, E.D., 2006. Statistical inference using the G or K point pattern spatial statistics. Ecology 87, 1925–1931.
- Ripley, B.D., 1977. Modelling of spatial patterns. J. R. Stat. Soc. B 39, 172–192.

Thank you!