Modeling group dispersal of particles with a spatiotemporal point process

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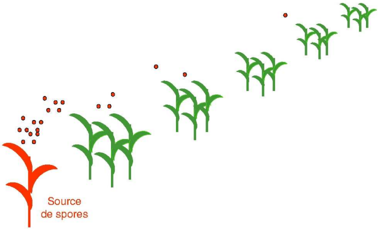
Group Dispersal Project – Plant Health and Environment Dpt.

Spatiotemporal point processes in propagation models

Object of interest: species spreading using small particles (spores, pollens, seeds...)



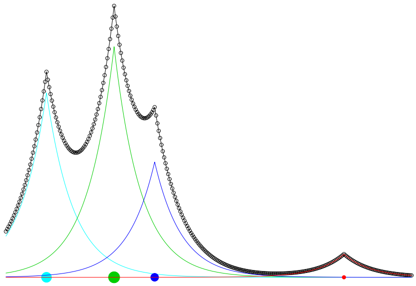
Sources of particles generate a spatially structured *rain* of particles

- rain of particles $\rightarrow$ spatial point process
- spatial structure $\rightarrow$ inhomogeneous intensity of the process

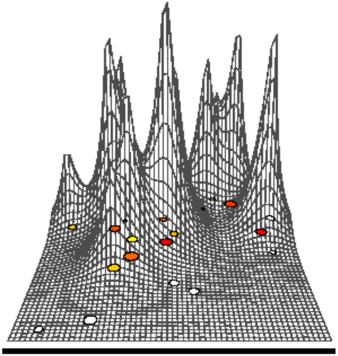
Intensity of the spatial point process formed by the deposit locations of the particles

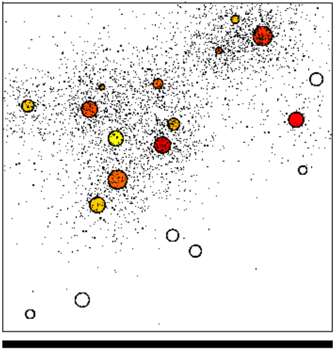
The intensity is a convolution between

- the source process (spatial pattern and strengths) and
- a parametric dispersal kernel



Simulation of an epidemics





Dispersal kernel

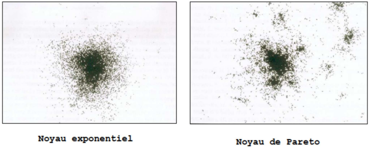
Dispersal kernel: probability density function of the deposit location of a particle released at the origin

The shape of the kernel is a major topic in dispersal studies: it determines

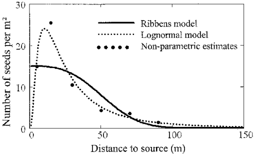
- the propagation speed
- the spatial structure of the population
- the genetic structure of the population

Main characteristics of dispersal kernels:

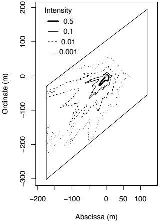
- long distance dispersal (Minogue, 1989)



- non-monotonicity (Stoyan and Wagner, 2001)

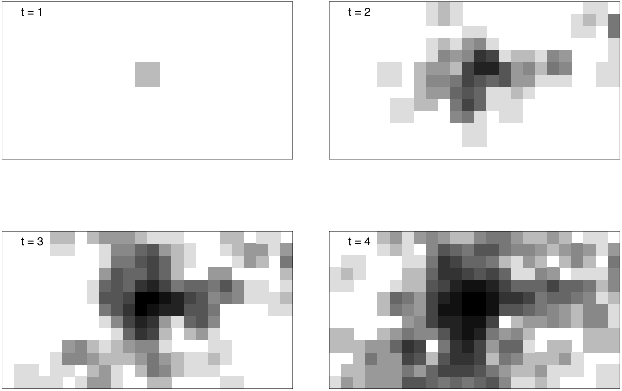


- anisotropy



Observation of secondary foci (clusters) in real epidemics

Epidemics of yellow rust of wheat in an experimental field (I. Sache)



- Classical justifications for patterns with multiple foci:
  - long distance dispersal
  - spatial heterogeneity
  - super-spreaders (a few individuals which infects many susceptible individuals)

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  - spatial heterogeneity
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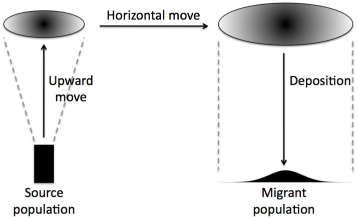
- An other justification to be investigated: **Group dispersal**
  - Groups of particles are released due to wind gusts
  - Particles of any group are transported in an expanding air volume
  - At a given stopping time, particles of any group are projected to the ground



Group Dispersal Model (GDM): Spatial case

Deposit equation for particles:

A single point source of particles located at the origin of $\mathbb{R}^2$

J: number of groups of particles released by the source

N_j: number of particles in group $j \in \{1, \ldots, J\}$

X_{jn}: deposit location of the nth particle of group j satisfying

$$X_{jn} = X_j + B_{jn}(\nu ||X_j||), \tag{1}$$

where

X_j: final location of the center of group j,

B_{jn}: Brownian motion describing the relative movement of the nth particle in group j with respect to the group center

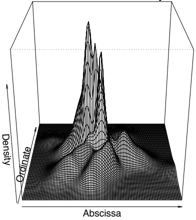
ν: positive parameter

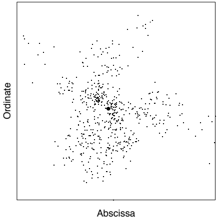
Assumptions about the deposit equation

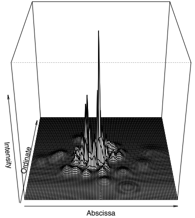
- The random variables J, N_j, X_j and the random processes $\{B_{jn} : n = 1, \ldots, N_j\}$ are mutually independent
- Number of groups: $J \sim Poisson(\lambda)$
- Number of particles in group j: $N_j \sim_{indep} p_{\mu,\sigma^2}(\cdot)$
- Group center location: $X_j \sim_{indep} f_{X_j}(\cdot)$
  (features of f_{X_j}: decrease at the origin is more or less steep, tail more or less heavy, shape more or less anisotropic...)
- The Brownian motions B_{jn} are centered, independent and with independent components
  They are stopped at time $t = \nu||X_j||$.
  Then,
  $$B_{jn}(\nu||X_j||) \sim_{indep} N(0, \nu||X_j||I)$$

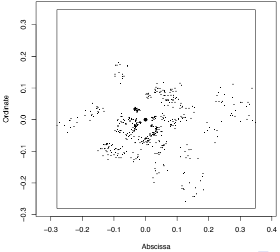
Dispersal from a single source

- Simulations: (Interpretation: Cox process or Neyman-Scott with double nonstationarity — in the center pattern and the offspring diffusion)









Dispersal from a single source

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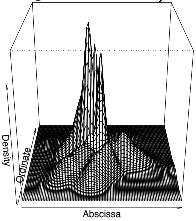
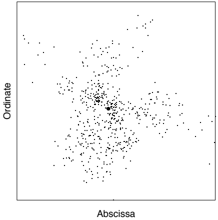




- Marginal probability density function (dispersal kernel):

$$f_{X_{jn}}(x) = \int_{\mathbb{R}^2} f_{X_{jn}|X_j}(x \mid y) f_{X_j}(y) dy = \int_{\mathbb{R}^2} \phi_{\nu,y}(x) f_{X_j}(y) dy.$$

The particles are n.i.i.d. from this p.d.f. while in the classical dispersal models the particles are i.i.d. from a dispersal kernel which may be of the form of f_{X_j} or $f_{X_{jn}}$

Discrepancies from independent dispersal

The GDM is compared with two independent dispersal models (IDM)

- IDM1: the number of particles in each group is assumed to be one. Thus, particles are independently drawn under the p.d.f. $f_{X_{jn}}$.
- IDM2: the number of particles in each group is assumed to be one and the Brownian motions are deleted (i.e. $\nu = 0$). Thus, particles are independently drawn under the p.d.f. f_{X_j}.

Moments

X: Deposit location of a particle

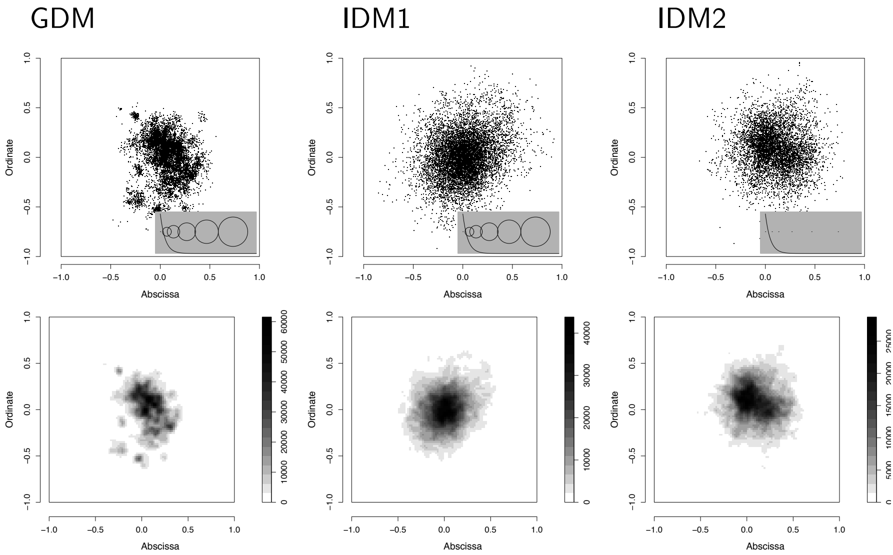
$Q(x+dx)$: Count of points in $x+dx$

Criterion	Model	Value
$E(X)$	GDM	$\binom{0}{0}$
	IDM1	$\binom{0}{0}$
	IDM2	$\binom{0}{0}$
$V(X)$	GDM	$V(X_j) + \nu E(\vert\vert X_j\vert\vert)I$
	IDM1	$V(X_j) + \nu E(\vert\vert X_j\vert\vert)I$
	IDM2	$V(X_j)$
$E(\vert\vert X\vert\vert^2)$	GDM	$E(\vert\vert X_j\vert\vert^2) + 2\nu E(\vert\vert X_j\vert\vert)$
	IDM1	$E(\vert\vert X_j\vert\vert^2) + 2\nu E(\vert\vert X_j\vert\vert)$
	IDM2	$E(\vert\vert X_j\vert\vert^2)$
$E\{Q(x+dx)\}$	GDM	$\lambda\mu f_{X_{jn}}(x)dx$
	IDM1	$\lambda f_{X_{jn}}(x)dx$
	IDM2	$\lambda f_{X_j}(x)dx$
$V\{Q(x+dx)\}$	GDM	$\lambda[\mu f_{X_{jn}}(x)dx + (\sigma^2+\mu^2-\mu)E\{\phi_{\nu,X_j}(x)^2\}(dx)^2]$
	IDM1	$\lambda f_{X_{jn}}(x)dx$
	IDM2	$\lambda f_{X_j}(x)dx$
$\mathrm{cov}\{Q(x_1+dx)$	GDM	$\lambda(\sigma^2+\mu^2-\mu)E\{\phi_{\nu,X_j}(x_1)\phi_{\nu,X_j}(x_2)\}(dx)^2$
$, Q(x_2+dx)\}$	IDM1	0
	IDM2	0

GDM: larger variance of $Q(x + dx)$ and positive covariance (decreasing with distance)
$\rightarrow$ clusters (even with $\mu = 1$)

We expect multiple foci in the spatio-temporel case

Group dispersal model: Spatio-temporal case

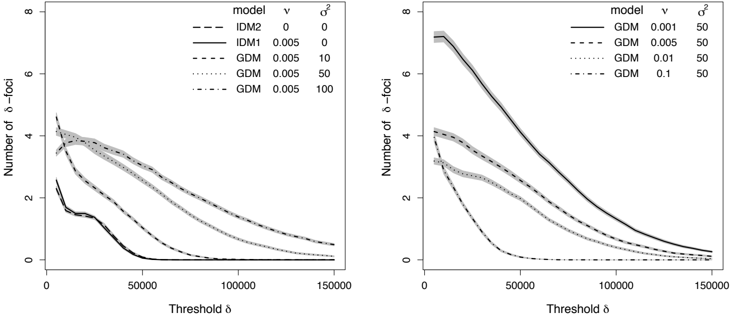


→ multiple foci under the GDM

Simulation study of the number of foci:

Definition

A δ-focus is a set of cells (from a regular grid) which are connected and whose intensity of points is larger than δ



Farthest particle (link with propagation speed)

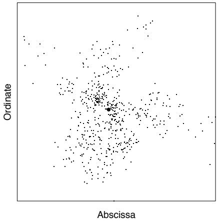
Definition

The maximum dispersal distance during one generation is

$$R^{max} = \max\{R_{jn} : j \in \mathcal{J}, n \in \mathcal{N}_j\}$$

where $R_{jn} = ||X_{jn}||$
$\mathcal{J} = \{1, \ldots, J\}$ if $J > 0$ and the empty set otherwise
$\mathcal{N}_j = \{1, \ldots, N_j\}$ if $N_j > 0$ and the empty set otherwise

By convention, if no particle is released ($J = 0$ or $N_j = 0$ for all j), then $R^{max} = 0$



$$R^{max} = \max\{R_{jn} : j \in \mathcal{J}, n \in \mathcal{N}_j\}$$

Under the GDM and IDMs, the distribution of the distance between the origin and the furthest deposited propagule is zero-inflated and satisfies:

$$P(R^{max} = 0) = \exp\left[\lambda\{p_{\mu,\sigma^2}(0) - 1\}\right]$$
$$f_{R^{max}}(r) = \lambda f_{R_j^{max}}(r) \exp\{\lambda(F_{R_j^{max}}(r) - 1)\}, \qquad \forall r > 0,$$

where $f_{R_j^{max}}$ is the p.d.f. of the distance $R_j^{max} = \max\{R_{jn} : n \in \mathcal{N}_j\}$ between the origin and the furthest deposited propagule of group j, and $F_{R_j^{max}}$ is the corresponding cumulative distribution function ($F_{R_j^{max}}(r) = P(R_j^{max} = 0) + \int_0^r f_{R_j^{max}}(u)du$).

$\rightarrow$ Distribution of R_j^{max} ?

Under the IDMs, $N_j = 1$ for all $j \in \mathcal{J}$ and, consequently, $p_{\mu,\sigma^2}(0) = 0$ and

$$f_{R_j^{max}}(r) = f_{R_{jn}}(r)$$
$$= \begin{cases} \int_0^{2\pi} r f_{X_{jn}}((r\cos\theta, r\sin\theta))d\theta & \text{for the IDM1} \\ \int_0^{2\pi} r f_{X_j}((r\cos\theta, r\sin\theta))d\theta & \text{for the IDM2.} \end{cases}$$

Under the GDM, the distribution of R_j^{max} is zero-inflated and satisfies:

$$P(R_j^{max} = 0) = p_{\mu,\sigma^2}(0)$$

$$\begin{aligned} f_{R_j^{max}}(r) &= \int_{\mathbb{R}^2} f_{R_j^{max}|X_j}(r \mid x) f_{X_j}(x)dx \\ &= \sum_{q=1}^{+\infty} q p_{\mu,\sigma^2}(q) \int_{\mathbb{R}^2} f_{R_{jn}|X_j}(r \mid x) F_{R_{jn}|X_j}(r \mid x)^{q-1} f_{X_j}(x)dx, \end{aligned}$$

where $f_{R_{jn}|X_j}$ is the conditional distribution of R_{jn} given X_j satisfying

$$\begin{aligned} f_{R_{jn}|X_j}(r \mid x) &= 2r \int_0^{r^2} h_1(u,x) h_2(r^2 - u, x)du, \\ h_i(u,x) &= \frac{f_i(\sqrt{u}, x) + f_i(-\sqrt{u}, x)}{2\sqrt{u}}, \quad \forall i \in \{1,2\}, \\ f_i(v,x) &= \frac{1}{\sqrt{2\pi\nu||x||}} \exp\left(-\frac{(v - x^{(i)})^2}{2\nu||x||}\right), \quad \forall i \in \{1,2\}, \end{aligned}$$

$x = (x^{(1)}, x^{(2)})$ and $F_{R_{jn}|X_j}(r \mid x) = \int_0^r f_{R_{jn}|X_j}(s \mid x)ds$.

Theorem

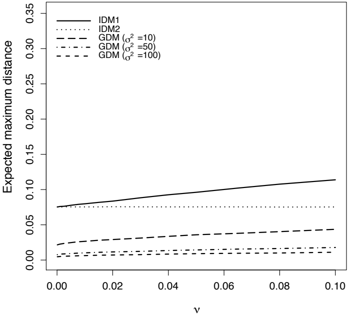
Consider a GDM and an IDM1 characterized by the same parameter values except that $E(J) = \tilde{\lambda}$, $E(N_j) = \tilde{\mu}$ *and* $V(N_j) = \sigma^2$ *for the GDM, and* $E(J) = \tilde{\lambda}\tilde{\mu}$, $E(N_j) = 1$ *and* $V(N_j) = 0$ *for the IDM1 (*$\Rightarrow$ *same marginal dispersal kernel).*
Then, for all $r > 0$ *the probability* $P(R^{max} \geq r)$ *is lower for the GDM than for the IDM1.*

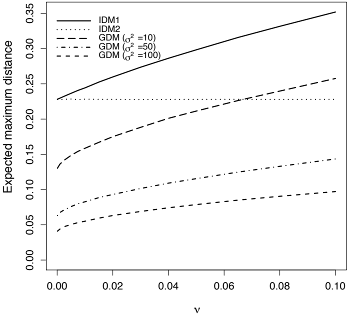
Theorem

Consider an IDM1 and an IDM2 characterized by the same parameter values except that $\nu > 0$ *for the IDM1 and* $\nu = 0$ *for the IDM2.*
Then, for all $r > 0$ *the probability* $P(R^{max} \geq r)$ *is lower for the IDM2 than for the IDM1.*

Interpretation:
The population of particles are less concentrated in probability for the IDM1 than for the GDM and the IDM2

$E(R^{max})$:


IDM1
IDM2
GDM (σ^2 =10)
GDM (σ^2 =50)
GDM (σ^2 =100)
Expected maximum distance
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.00
0.02
0.04
0.06
0.08
0.10
ν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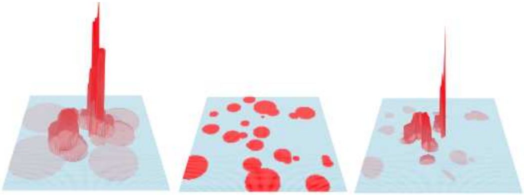
0.02
0.04
0.06
0.08
0.10
ν

Conclusion

- With group dispersal, one can generate multiple foci whereas the particles are more concentrated

Perspectives

- Toward analytic results about the farthest particle in the spatio-temporal case
  ($\rightarrow$ speed of propagation of epidemics)
- Inference (with Tomas Mrkvicka and Eyoub Sidi)
- Alternative representations of group dispersal (Cylinder-based models, with Tomas Mrkvicka and Antti Penttinen)



- Study of the evolutionary dynamics between group dispersal and independent dispersal