Identification of local multivariate outliers

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Introduction

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Objective : identify/detect

- gross errors,
- atypical observations

taking into account the multivariate and the spatial nature of the data.

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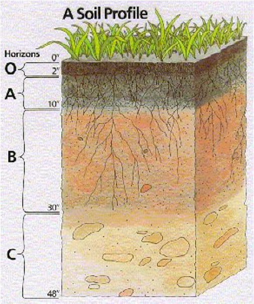
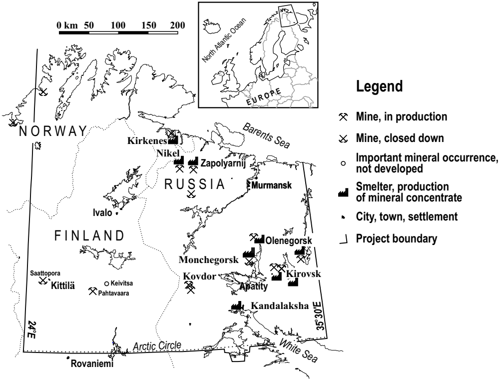
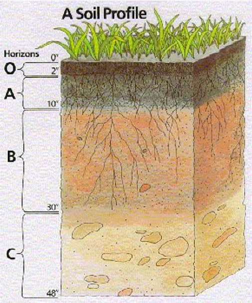


The Kola project : concentration measures for more than 50 chemical elements in four layers and 617 observations.





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Data available in the R-package mvoutlier by M. Gschwandtner et P. Filzmoser.

1. Detection of outliers in a non spatial context
   - Detection of univariate outliers
   - Detection of multivariate outliers

2. Spatial outliers
   - Global and local outliers
   - Identification of univariate spatial outliers

3. Identification of multivariate spatial outliers
   - Variocloud of pairwise Mahalanobis distances
   - Toy example
   - Quantile geographical-variate plot

Detection of univariate outliers

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Robust version : $\bar{x}$ and σ_x replaced by some robust estimators such as the median and the MAD.

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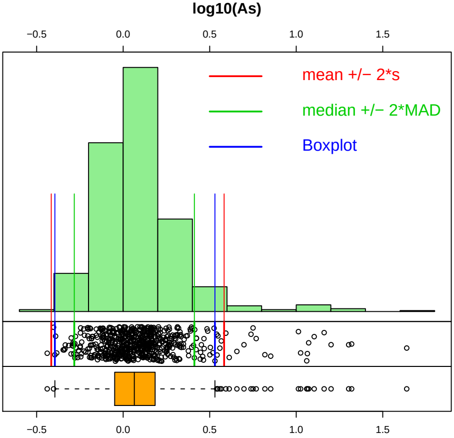

FIG.: Histogram of log(Arsenic)

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In the Gaussian case $\mathcal{N}(\mu, \Sigma)$, the $\mathrm{MD}^2(x_i, \mu, \Sigma)$ follow a chi-square distribution with p degrees of freedom and a common used cut-off value is the quantile of order 95% of this chi-square distribution.

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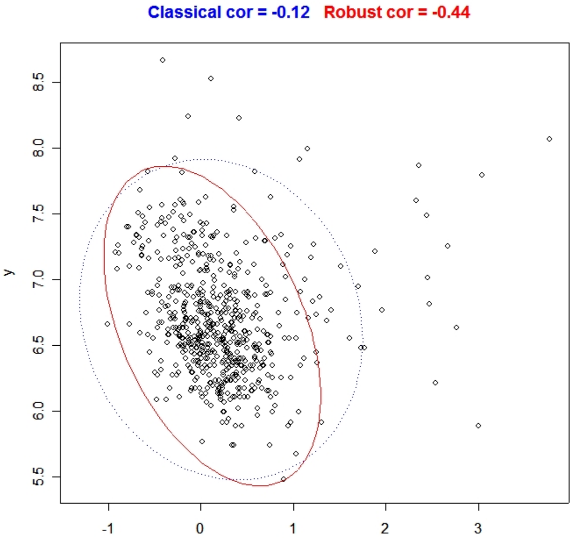
R-packages mvoutliers et rrcov.

In two dimensions, scatterplot with ellipsoids corresponding to non-robust estimators (blue) t and C and MCD estimators (red) for a quantile of order 95%.



Global and local outliers

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Local outlier : relative to the sub-distribution associated with the observation and its neighborhood.
Underlying assumption : positive spatial autocorrelation.

Exploratory plots for identifying univariate spatial outliers

- Neighbor plot
- Moran plot
- Drift map
- Angle plot
- Variocloud

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R-package GeoXp by C. Thomas et al. (2012).

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Draw only part of the cloud, summarize the rest by conditional quantile curves

Example of multivariate variocloud

Selected units : small spatial distances and high pairwise Mahalanobis distance

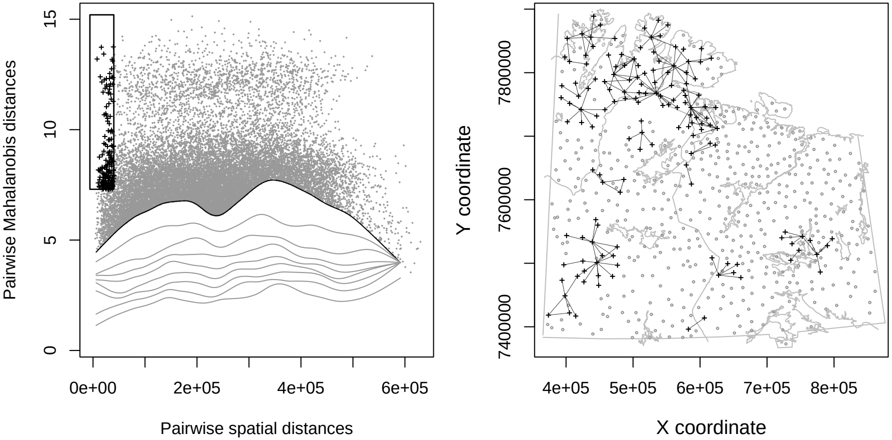


Fig. : Multivariate variocloud

A small toy example

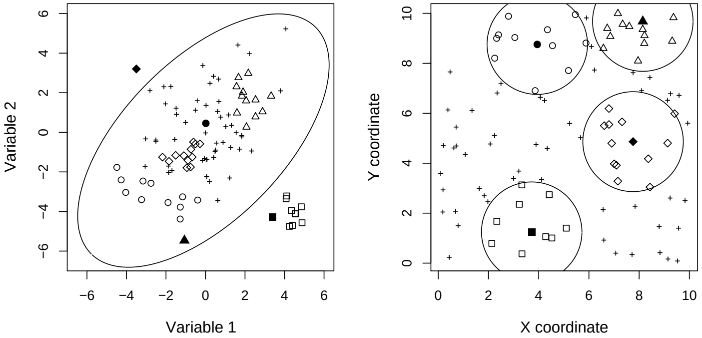
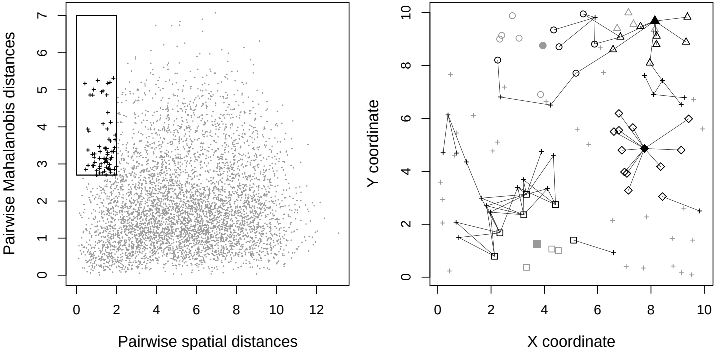


FIG.: Toy example

Comparing pairwise geographical distance and pairwise distance in the non spatial attributes space.

Variocloud for the toy example



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Distribution of the pairwise Mahalanobis Distance

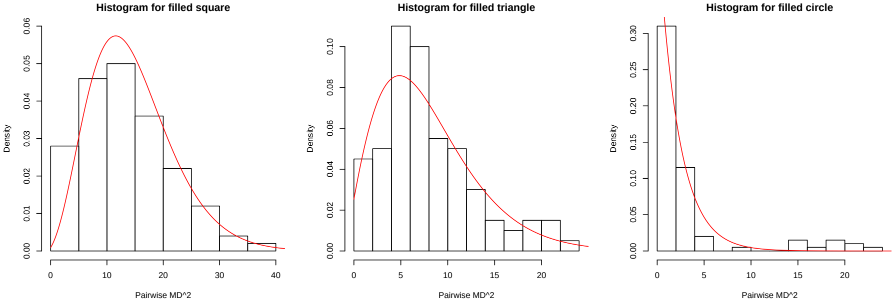
When the observations $X_1, \ldots, X_n$ are i.i.d. with a normal distribution $\mathcal{N}(\mu, \Sigma)$, we can prove that :

Conditional on one observation X_i, the distribution of the squared pairwise Mahalanobis distance

$\mathrm{MD}^2(X_i, X_j, \Sigma)$ of X_j, $j \neq i$ is a non central chi-square distribution

with the Mahalanobis distance $\mathrm{MD}^2(X_i, \mu, \Sigma)$ as the non-centrality parameter and p degrees of freedom.

Distribution of pairwise MD : toy example



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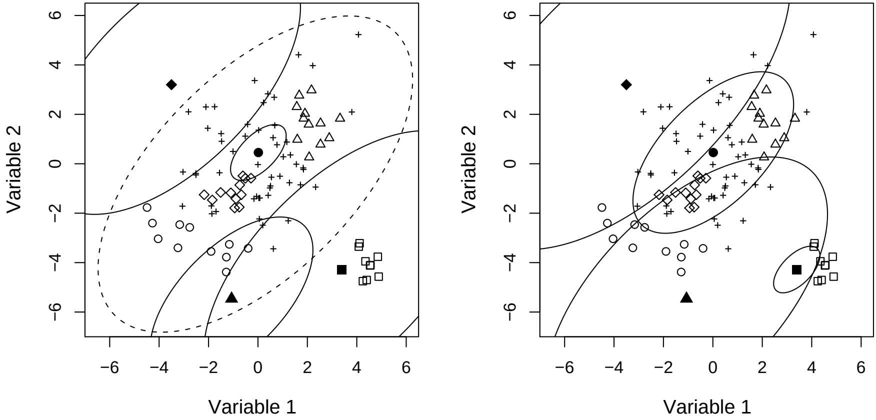
Quantile geographical-variate plot



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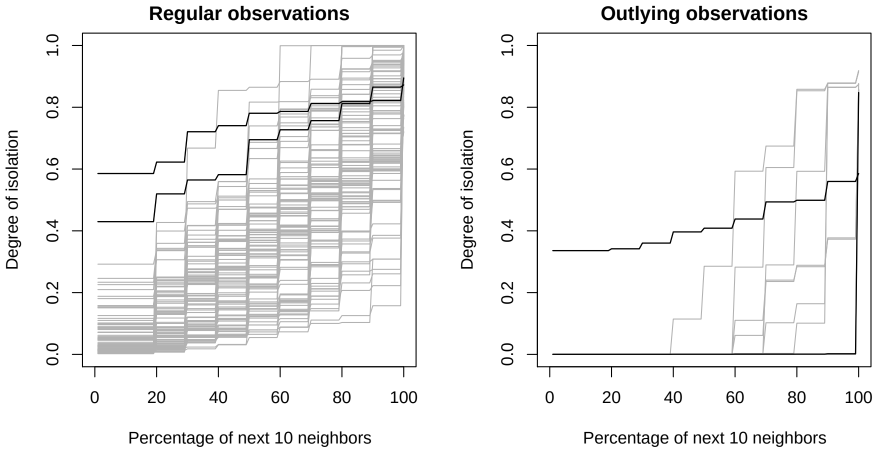

FIG.: Toy example

Quantile geographical-variate plot on Kola example

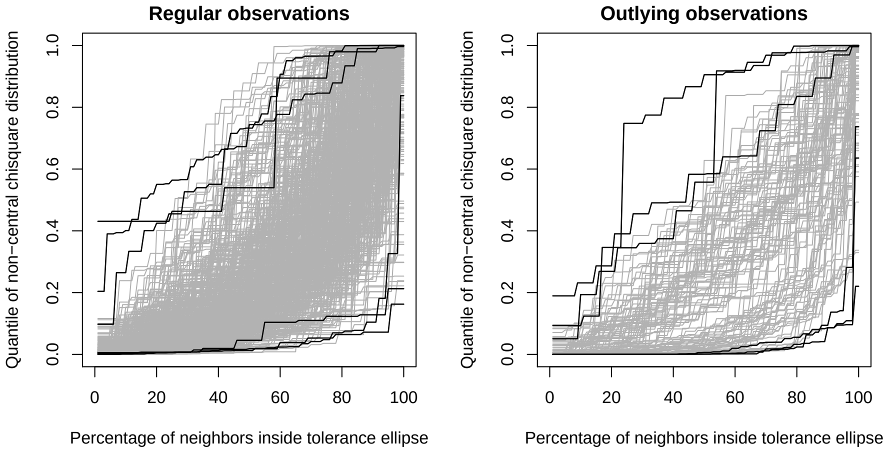

FIG.: Kola example

Conclusion

Thank you for your attention !